INTUITIONISTIC FUZZY NUMBER GRACEFUL LABELING GRAPHS

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Abstract: Certain idea about the intuitionistic fuzzy number graceful labeling graph of a intuitionistic fuzzy graph is given and some related concepts of the intuitionistic fuzzy number graceful labeling graph are also discussed.

Keywords: Fuzzy set, fuzzy graph, fuzzy number graceful labeling, intuitionistic fuzzy set, intuitionistic fuzzy graph, intuitionistic fuzzy number graceful labeling, intuitionistic fuzzy cycle graph, intuitionistic fuzzy path graph.

INTRODUCTION

First, fuzzy set had been introduced by Zadeh [18]. Succeeding years, fuzzy set was grown in different ways. The following are extension of fuzzy set, they are vague set, intuitionistic fuzzy set, bipolar valued fuzzy set and etc. In 1975, fuzzy graph was introduced by Rosenfeld [12], with modification of fuzzy graph was introduced by K.Arjunan and C.Subramani[2] and it was extended to many area. In similar way, [3], [4], [5], [6], [7], [8], [9], [10], [11], [13], [14], [15], [16] and [17] were useful to write this paper. K.Arjunan et all.[1] have given an idea about the fuzzy number graceful labeling graph. In this paper, graceful labeling is generalized, particularly intuitionistic fuzzy number graceful labeling, the triangular intuitionistic fuzzy number is used in this paper but we have different types of intuitionistic fuzzy number.

1. PRELIMINARIES

Definition 1.1. [18] A fuzzy set Ñ on the given universal set Q is a set of ordered pairs Ñ = {(x, $F_m(x)$): $x \in$Q}, where F_m: Q → [0,1], is called membership function.

Definition 1.2. [18] The k − cut of a fuzzy set Ñ, is defined by ÑL = {z: $F_m(z) \geq k$}, where k∈[0, 1].

Definition 1.3. [18] Let Φ, Θ be two fuzzy subsets of a set G. The following relations and operations are defined as:

(i) $\Phi \subset \Theta$ means $\Phi(t) \leq \Theta(t)$, for all t∈G.

(ii) $\Phi = \Theta$ means $\Phi(t) = \Theta(t)$, for all t∈G.

(iii) $\Phi^c(t) = 1 - \Phi(t)$, for all t∈G.

(iv) $\Phi\cap\Theta = \{\langle t, \min(\Phi(t), \Theta(t)) \rangle / t\in G\}$.

(v) $\Phi\cup\Theta = \{\langle t, \max(\Phi(t), \Theta(t))\rangle / t\in G\}$.

Definition 1.4. [2] Let Φ be a FS in a set G. The strongest fuzzy relation Ψ is an ordered pair Ψ = {⟨(c_1, d_1), Ψ(c_1, d_1)⟩ / c_1, d_1∈G} on G which is a fuzzy relation on Φ and is defined by Ψ(c_1, d_1) = min{Φ(c_1), Φ(d_1)} for all c_1, d_1∈G.

Definition 1.5. [2] Let S be any nonempty set and L be any set with a function ℑ: L→S×S. Then Φ is a $_F$S of S, R is a fuzzy relation on S with respect to Φ and Θ is a $_F$S of L such that $\Theta(e) \le \underset{e\in\Im^{-1}(x,y)}{R}(x,y)$. Then the ordered triple Ω = (Φ, Θ, ℑ) is called a fuzzy graph ($_F$G), here the elements of Φ are fuzzy points or fuzzy vertices ($_F$V) and the elements of Θ are fuzzy lines or fuzzy edges ($_F$E) of the $_F$G Ω. Let ℑ(a) = (c_1, d_1). Then (c_1, Φ(c_1)), (d_1, Φ(d_1)) are adjacent $_F$Vs and (c_1, Φ(c_1)), $_F$E (a, Θ(a)) are incident with each other. Let two distinct $_F$Es (a_1, Θ(a_1)) and (a_2, Θ(a_2)) are incident with a common $_F$V. They are called adjacent $_F$Es. Let Φ^* = {w∈S / Φ(w) > 0} and Θ^* = {c∈L / Θ(c) > 0. Then Ω^* = (Φ^*, Θ^*, ℑ) is a crisp graph.

Definition 1.6. [1] A fuzzy number graceful labeling($_{FN}$GfL) of a $_F$G Ω = (Φ, Θ, ℑ) having q $_F$Es is an injective mapping 𝔏 : Φ(Ω) → { $\tilde{0}$, $\tilde{1}$, $\tilde{2}$, . . . , $\tilde{q}$} such that when each $_F$E $\zeta\varsigma$ is assigned the label |𝔏(ζ) − 𝔏(ς) |, the resulting $_F$E labels are distinct. A fuzzy number graceful graph($_{FN}$GfG) is one which admits a $_{FN}$GfL.

2. INTUITIONISTIC FUZZY GRAPH

Definition 2.1. [3] Let V be any nonempty set, E be any set and f : E→V×V be any function. Then A is an $_{IF}$S of V, S is an $_{IF}$ relation on V with respect to A and B is an $_{IF}$S of E such that $\mu_B(e) \le \underset{e\in f^{-1}(x,y)}{\mu_S}(x,y)$ and $\gamma_B(e) \ge \underset{e\in f^{-1}(x,y)}{\gamma_S}(x,y)$. Then the ordered triple F = (A, B, f) is called an **intuitionistic fuzzy graph($_{IF}$G)**, where the elements of A are called **intuitionistic fuzzy points** or **intuitionistic fuzzy vertices($_{IF}$Vs)** and the elements of B are called **intuitionistic fuzzy lines** or **intuitionistic fuzzy edges($_{IF}$Es)** of the $_{IF}$G F. If f(e) = (x, y), then the $_{IF}$Vs (x, μ_A(x), γ_A(x)), (y, μ_A(y), γ_A(y)) are called **adjacent** $_{IF}$**Vs** and $_{IF}$Vs (x, μ_A(x), γ_A(x)), $_{IF}$E (e, μ_B(e), γ_B(e)) are called **incident** with each other. If two distinct $_{IF}$Es (e_1, μ_B(e_1), γ_B(e_1)) and (e_2, μ_B(e_2), γ_B(e_2)) are incident with a common $_{IF}$V, then they are called **adjacent** $_{IF}$**Es.**

Definition 2.2. [3] An $_{IF}$E joining an $_{IF}$V to itself is called an **intuitionistic fuzzy loop($_{IF}$L)**.

Definition 2.3. [3] Let F = (A, B, f) be an $_{IF}$G. If two or more $_{IF}$Es of F have the same $_{IF}$Vs, then these $_{IF}$Es are called **intuitionistic fuzzy multiple edges($_{IF}$MEs)**. An $_{IF}$G which has $_{IF}$MEs are called an **intuitionistic fuzzy multigraph($_{IF}$MG).**

Definition 2.4. [3] Let F = (A, B, f) be an $_{IF}$G. If both $_{IF}$Ls and $_{IF}$MEs, then the $_{IF}$G F is called an **intuitionistic fuzzy pseudo graph($_{IF}$PG)**.

Definition 2.5. [3] An $_{IF}$G F = (A, B, f) is called an **intuitionistic fuzzy simple graph($_{IF}$SG)** if it has neither $_{IF}$MEs nor $_{IF}$Ls.

Example 2.6. Let V = {v_1, v_2, v_3, v_4, v_5 } and E = {a, b, c, d, e, h, g} be two non-empty sets. The function f : E → V×V is defined by f(a) = (v_1, v_2), f(b) = (v_2, v_2), f(c) = (v_2, v_3), f(d) = (v_3, v_4), f(e) = (v_3, v_4), f(h) = (v_4, v_5), f(g) = (v_1, v_5). An $_{IF}$S A = { (v_1, 0.8, 0.1), (v_2, 0.7, 0.2), (v_3, 0.8, 0.2), (v_4, 0.9, 0.1), (v_5, 0.5, 0.2) } of V, $_{IF}$R S = {((v_1, v_1), 0.8, 0.1), ((v_1, v_2), 0.7, 0.2), ((v_1, v_3), 0.8, 0.2), ((v_1, v_4), 0.8, 0.1), ((v_1, v_5), 0.5, 0.2), ((v_2, v_1), 0.7, 0.2), ((v_2, v_2), 0.7, 0.2), ((v_2, v_3), 0.7, 0.2), ((v_2, v_4), 0.7, 0.2), ((v_2, v_5), 0.5, 0.2), ((v_3, v_1), 0.8, 0.2), ((v_3, v_2), 0.7, 0.2), ((v_3, v_3), 0.8, 0.2), ((v_3, v_4), 0.8, 0.2), ((v_3, v_5), 0.5, 0.2), ((v_4, v_1), 0.8, 0.1), ((v_4, v_2), 0.7, 0.2), ((v_4, v_3), 0.8, 0.2), ((v_4, v_4), 0.9, 0.1), ((v_4, v_5), 0.5, 0.2), ((v_5, v_1), 0.5, 0.2), ((v_5, v_2), 0.5, 0.2), ((v_5, v_3), 0.5, 0.2), ((v_5, v_4), 0.5, 0.2), ((v_5, v_5), 0.5, 0.2)} on V with respect to A and the $_{IF}$S B = {(a, 0.7, 0.2), (b, 0.6, 0.2), (c, 0.6, 0.3), (d, 0.7, 0.2), (e, 0.8, 0.2), (h, 0.4, 0.3), (g, 0.3, 0.2)} of E. Then F = (A, B, f) is an IFG.

s

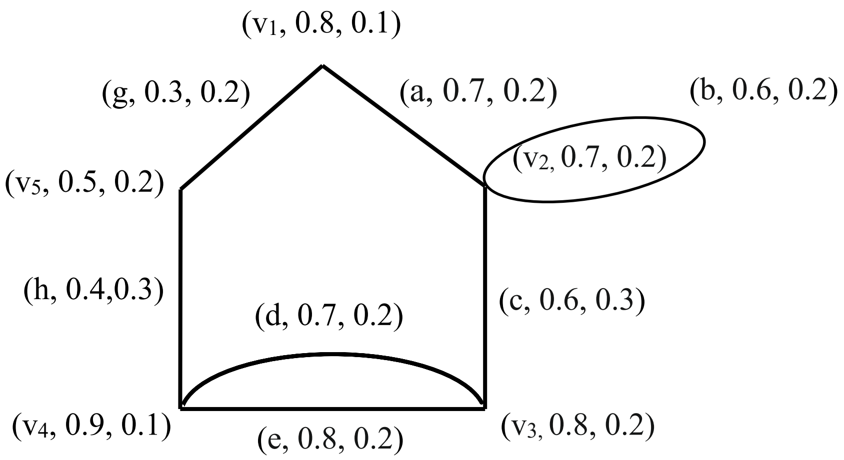

Fig. 2.1

Definition 2.7. [3] An IFG H = (C, D, f) where C and D is called an **intuitionistic fuzzy subgraph(IFSubG)** of F = (A, B, f) if C $\subseteq$ A and D $\subseteq$ B.

Definition 2.8. [3] An IFSubG H = (C, D, f) is said to be an **intuitionistic fuzzy spanning subgraph(IFSSubG)** of F = (A, B, f) if C = A.

Definition 2.9. [3] An IFSubG H = (C, D, f) is said to be an **intuitionistic fuzzy induced subgraph(IFISubG)** of F = (A, B, f) if H is the maximal IFSubG of F with IFV set C.

Definition 2.10. [3] Let F = (A, B, f) be an IFG with respect to the sets V and E. Let C be an IFS of V and C $\subseteq$ A. Then IFS D of E is defined as $\mu_D(e) = \min\{\mu_C(u), \mu_C(v), \mu_B(e)\}$, $\gamma_D(e) = \max\{\gamma_C(u), \gamma_C(v), \gamma_B(e)\}$ for all e in E. Then H = (C, D, f) is called **intuitionistic fuzzy partial subgraph(IFPSubG)** of F.

Definition 2.11. [3] Let F = (A, B, f) be an IFG. Let (x, $\mu_A(x)$, $\gamma_A(x)$) be an IFV of F. The IFSubG of F obtained by removing the IFV (x, $\mu_A(x)$, $\gamma_A(x)$) and all the IFEs incident with (x, $\mu_A(x)$, $\gamma_A(x)$) is called the IFSubG obtained by the removal of the IFV (x, $\mu_A(x)$, $\gamma_A(x)$) and is denoted F− (x, $\mu_A(x)$, $\gamma_A(x)$). If F− (x, $\mu_A(x)$, $\gamma_A(x)$) = (C, D, f) then C = A − {(x, $\mu_A(x)$, $\gamma_A(x)$) } and D = { (e, $\mu_B(e)$, $\gamma_B(e)$) / (e, $\mu_B(e)$, $\gamma_B(e)$) $\in$ B and (x, $\mu_A(x)$, $\gamma_A(x)$) is not incident with (e, $\mu_B(e)$, $\gamma_B(e)$) }. Clearly F− (x, $\mu_A(x)$, $\gamma_A(x)$) is **intuitionistic fuzzy induced subgraph** of F. If (e, $\mu_B(e)$, $\gamma_B(e)$) $\in$ B then F − (e, $\mu_B(e)$, $\gamma_B(e)$) = (A, D, f) is called IFSubG of F obtained by the removal of the IFE (e, $\mu_B(e)$, $\gamma_B(e)$), where D = B −{(e,$\mu_B(e)$,$\gamma_B(e)$) }. Clearly F − (e, $\mu_B(e)$, $\gamma_B(e)$) is an IFSSubG of F which contains all the lines of F except (e, $\mu_B(e)$, $\gamma_B(e)$).

Definition 2.12. [3] From an IFG F, delete the all IFLs and delete the more than one IFEs from the IFMEs, this type of graph is called an **intuitionistic fuzzy underling simple graph(IFUSG) of F.**

Definition 2.13. [3] Let F = (A, B, f) be an IFG. Then the **degree of an intuitionistic fuzzy vertex** is defined by d(v) = ($d_\mu(v)$, $d_\gamma(v)$) where $d_\mu(v) = \sum_{e\in f^{-1}(u,v)} \mu_B(e) + 2 \sum_{e\in f^{-1}(v,v)} \mu_B(e)$ and $d_\gamma(v) = \sum_{e\in f^{-1}(u,v)} \gamma_B(e) + 2 \sum_{e\in f^{-1}(v,v)} \gamma_B(e)$.

Definition 2.14. [3] The **minimum degree** of the IFG F = (A, B, f) is $\delta(F) = (\delta_\mu(F), \delta_\gamma(F))$ where $\delta_\mu(F) = \min\{d_\mu(v) / v\in V\}$ and $\delta_\gamma(F) = \min\{ d_\gamma(v) / v\in V\}$ and the **maximum degree** of F is $\Delta(F) = (\Delta_\mu(F), \Delta_\gamma(F))$ where $\Delta_\mu(F) = \max\{ d_\mu(v) / v\in V \}$ and $\Delta_\gamma(F) = \max\{ d_\gamma(v) / v\in V\}$.

Definition 2.15. [3] Let F = (A, B, f) be an IFG. Then the **order of intuitionistic fuzzy graph** F is defined to be $o(F) = (o_\mu(F), o_\gamma(F))$ where $o_\mu(F) = \sum_{v\in V}\mu_A(v)$ and $o_\gamma(F) = \sum_{v\in V}\gamma_A(v)$.

Definition 2.16. [3] Let F = (A, B, f) be an IFG. Then the **size of the intuitionistic fuzzy graph F** is defined to be $S(F) = (S_\mu(F), S_\gamma(F))$, where $S_\mu(F) = \sum_{e\in f^{-1}(u,v)}\mu_B(e)$ and $S_\gamma(F) = \sum_{e\in f^{-1}(u,v)}\gamma_B(e)$.

Example 2.17

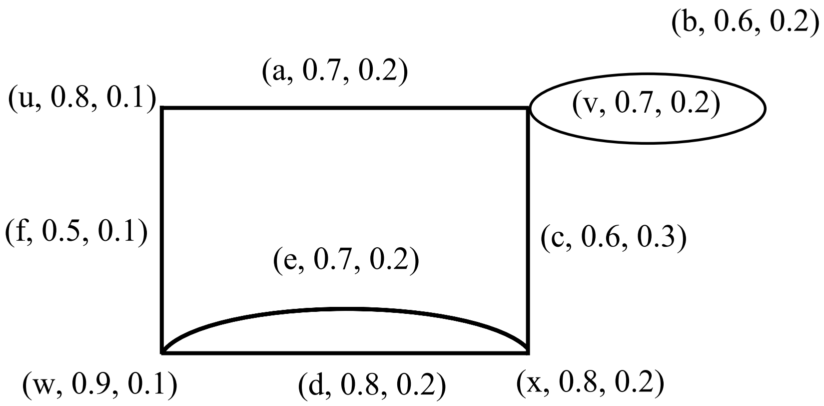

Fig. 2.2. IFG F

Here d(u) = (1.2, 0.3), d(v) = (2.5, 0.9), d(w) = (2, 0.5), d(x) = (2.1, 0.7), δ(F) = (1.2, 0.3), Δ(F) = (2.5, 0.9), o(F) = (3.2, 0.6), S(F) = (3.9, 1.2).

Definition 2.18. [3] An intuitionistic fuzzy path(IFPa) in an IFG F = (A, B, f) is an alternating sequence of IFVs and IFEs $(v_0, \mu_A(v_0), \gamma_A(v_0)),(e_1, \mu_B(e_1), \gamma_B(e_1)),(v_1, \mu_A(v_1), \gamma_A(v_1)), (e_2, \mu_B(e_2), \gamma_B(e_2)), (v_2, \mu_A(v_2), \gamma_A(v_2)),\ldots,(v_{n-1}, \mu_A(v_{n-1}), \gamma_A(v_{n-1})), (e_n, \mu_B(e_n), \gamma_B(e_n)), (v_n, \mu_A(v_n), \gamma_A(v_n))$ such that $B(e_i) = (\mu_B(e_i), \gamma_B(e_i)) > 0$, i = 1,2,…,n and all the IFVs are distinct. Here n is called the length of the IFPa. The strength of the IFPa P is defined as $\bigcap_{i=1}^{n} B(e_i)$. In other words the strength of P is defined to be the weight of the weakest IFE of the IFPa. A single IFV v may also be considered as an IFPa. In this case, the IFPa is of length 0. If an IFPa has length 0, its strength is A(v).

Definition 2.19. [3] An IFPa F = (A, B, f) is intuitionistic fuzzy connected graph(IFConG) if any two IFVs are joined by an IFPa.

Definition 2.20. [3] Let F = (A, B, f) be an IFG. Then F is an **intuitionistic fuzzy irregular(IFIR)** if there is an IFV which is adjacent to IFVs with distinct degrees.

Definition 2.21. [5] A intuitionistic fuzzy subset Ñ = {(ƨ, $Đ_ñ^+$(ƨ), $Đ_ñ^-$(ƨ)): ƨ∈R} is called generalized intuitionistic fuzzy number if

(i) $Đ_ñ^+$ is a continuous mapping from R to the closed interval [0, 1],

(ii) $Đ_ñ^+ = 0$, $-\infty <$ ƨ $\leq$ $ş_1$,

(iii) $Đ_ñ^+ = Ţ^+$(ƨ) is strictly increasing on [$ş_1$, $ş_2$],

(iv) $Đ_{ñ}^{+} = Ų^{+}_{Ƃ}$, $ȿ_2 \le ƨ \le ȿ_3$,

(v) $Đ_{ñ}^{+} = Ʀ^{+}(ƨ)$ is strictly decreasing on $[ȿ_3, ȿ_4]$,

(vi) $Đ_{ñ}^{+} = 0$, $ȿ_4 \le ƨ \le \infty$,

(vii) $Đ_{ñ}^{-}$ is a continuous mapping from R to the closed interval [0, 1],

(viii) $Đ_{ñ}^{-} = 1$, $-\infty < ƨ \le ȿ_0$,

(ix) $Đ_{ñ}^{-} = Ʈ^{-}(ƨ)$ is strictly decreasing on $[ȿ_0, ȿ_2]$,

(x) $Đ_{ñ}^{-} = Ų^{-}_{Ƃ}$, $ȿ_2 \le ƨ \le ȿ_3$,

(xi) $Đ_{ñ}^{-} = Ʀ^{-}(ƨ)$ is strictly increasing on $[ȿ_3, ȿ_5]$,

(xii) $Đ_{ñ}^{-} = 1$, $ȿ_5 \le ƨ \le \infty$,

where $0 < Ų_{Ƃ}^{+} + Ų^{-}_{Ƃ} \le 1$ and $ȿ_0, ȿ_1, ȿ_2, ȿ_3, ȿ_4, ȿ_5 \in R$, $ȿ_0 < ȿ_1 < ȿ_2 < ȿ_3 < ȿ_4 < ȿ_5$. It is denoted as $\tilde{N} = (ȿ_0, ȿ_1, ȿ_2, ȿ_3, ȿ_4, ȿ_5 ; Ų_{Ƃ}^{+}, Ų_{Ƃ}^{-})_{LR}$, if $Ų_{Ƃ}^{+} = 1$, $Ų_{Ƃ}^{-} = 0$, then $\tilde{N} = (ȿ_0, ȿ_1, ȿ_2, ȿ_3, ȿ_4, ȿ_5)_{ƮR}$.

Definition 2.22. [5] The intuitionistic fuzzy number $\tilde{N} = \{(ƨ, Đ_{ñ}^{+}(ƨ), Đ_{ñ}^{-}(ƨ)): ƨ\in R\} = ((\xi_1, \xi_2, \xi_3), (\xi_0, \xi_2, \xi_4))$ is called a triangular intuitionistic fuzzy number(T_{IFN}) if

$$Đ_{ñ}^{+} = \begin{cases} \dfrac{x-\xi_1}{\xi_2-\xi_1}, \xi_1 \le x \le \xi_2 \\ \dfrac{\xi_3-x}{\xi_3-\xi_2}, \xi_2 \le x \le \xi_3 \\ 0, otherwise \end{cases}$$

and

$$Đ_{ñ}^{-} = \begin{cases} \dfrac{\xi_2-x}{\xi_2-\xi_0}, \xi_0 \le x \le \xi_2 \\ \dfrac{x-\xi_2}{\xi_4-\xi_2}, \xi_2 \le x \le \xi_4 \\ 1, otherwise \end{cases}$$

Definition 2.23. [5] The collection $S = \{ \tilde{0}, \tilde{1}, \tilde{2}, \ldots \}$ of triangular intuitionistic fuzzy numbers is called non-negative triangular intuitionistic fuzzy numbers if $\tilde{a} = ((a_1,a,a_3), (a_0,a,a_4))$, $\tilde{a}\in S$, $a_0 \ge 0$. For example $\tilde{1} = ((0.7,1,2), (0.4,1,2.5))$ and $\tilde{2} = ((1.1,2,3), (1,2,3.2))$.

Definition 2.24. [5] Intuitionistic fuzzy arithmetical operations are defined for addition, subtraction, multiplication and division of triangular intuitionistic fuzzy numbers.

Let $\tilde{N} = ((\varsigma_1, \varsigma_2, \varsigma_3), (\varsigma_0, \varsigma_2, \varsigma_4))$ and $\hat{U} = ((\xi_1, \xi_2, \xi_3), (\xi_0, \xi_2, \xi_4))$ be two T_{IFN}s. Then (i) the addition of $\tilde{N}$ and $\hat{U}$ is $\tilde{N}+\hat{U} = ((\varsigma_1+\xi_1, \varsigma_2+\xi_2, \varsigma_3+\xi_3), (\varsigma_0+\xi_0, \varsigma_2+\xi_2, \varsigma_4+\xi_4))$,

(ii) The multiplication of $\tilde{N}$ and $\hat{U}$ is $\tilde{N}\times\hat{U} = ((\varsigma_1\xi_1, \varsigma_2\xi_2, \varsigma_3\xi_3), (\varsigma_0\xi_0, \varsigma_2\xi_2, \varsigma_4\xi_4))$, where $\varsigma_0, \varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \xi_0, \xi_1, \xi_2, \xi_3, \xi_4$ are all non zero positive real numbers,

(iii) –Û = ((−ξ3, −ξ2, −ξ1), (−ξ4, −ξ2, −ξ0)), the subtraction of Û from Ñ is Ñ – Û = ((ς_1 − ξ3, ς_2− ξ2, ς_3− ξ1), (ς_0− ξ4, ς_2− ξ2, ς_4− ξ0)),

(iv) $\frac{1}{\hat{U}} = \hat{U}^{-1} = ((\frac{1}{\xi_3}, \frac{1}{\xi_2}, \frac{1}{\xi_1}), (\frac{1}{\xi_4}, \frac{1}{\xi_2}, \frac{1}{\xi_0}))$, and the division of Ñ and Û is $\frac{\hat{N}}{\hat{U}} = ((\frac{\varsigma_1}{\xi_3}, \frac{\varsigma_2}{\xi_2}, \frac{\varsigma_3}{\xi_1}), (\frac{\varsigma_0}{\xi_4}, \frac{\varsigma_2}{\xi_2}, \frac{\varsigma_4}{\xi_0}))$ where ξ0, ξ1, ξ2, ξ3, ξ4 are all non zero positive real numbers,

(v) For any real number Ǩ,

ǨÑ = ((Ǩς_1, Ǩς_2, Ǩς_3),(Ǩς_0, Ǩς_2, Ǩς_4)) if Ǩ > 0

ǨÑ = ((Ǩς_3, Ǩς_2, Ǩς_1),(Ǩς_4, Ǩς_2, Ǩς_0)) if Ǩ < 0.

Definition 2.25. [5] (i). Intuitionistic defuzzification of triangular intuitionistic fuzzy numbers Ñ = ((a_1, a_2, a_3), (a_0, a_2, a_4)) can be done by the signed distance(SD). It is defined as

$$d(\hat{N},0) = S(\hat{N}) = \frac{1}{4}\int_0^1 [a_1 + l(a_2 - a_1) + a_3 - l(a_3 - a_2) + a_2 - (1-l)(a_2 - a_0) + a_2 + (1-l)(a_4 - a_2)]dl$$

$$= \frac{a_1 + 2a_2 + a_3 + a_0 + 2a_2 + a_4}{8} = \frac{a_1 + 4a_2 + a_3 + a_0 + a_4}{8}.$$

(ii). Total Integral Value Method (TIVM) of Ñ is defined as

$$TI(\hat{N}) = \frac{1}{2}\int_0^1 a_1 + l(a_2 - a_1) + a_3 - l(a_3 - a_2) + a_2 - (1-l)(a_2 - a_0) + a_2 + (1-l)(a_4 - a_2)]dl$$

$$= \frac{a_1 + 2a_2 + a_3 + a_0 + 2a_2 + a_4}{4} = \frac{a_1 + 4a_2 + a_3 + a_0 + a_4}{4}.$$

(iii). The ranking Method (RM) of Ñ is defined as

$$R(\hat{N}) = \frac{a_3 - a_1}{a_4 - a_0}\frac{1}{4}\int_0^1 [a_1 + l(a_2 - a_1) + a_3 - l(a_3 - a_2) + a_2 - (1-l)(a_2 - a_0) + a_2 + (1-l)(a_4 - a_2)]dl$$

$$= \frac{a_3 - a_1}{a_4 - a_0}\frac{a_1 + 2a_2 + a_3 + a_0 + 2a_2 + a_4}{8} = \frac{a_3 - a_1}{a_4 - a_0}\frac{a_1 + 4a_2 + a_3 + a_0 + a_4}{8}.$$

Definition 2.26. [5] The intuitionistic fuzzy number Ñ = {(ƨ, $Đ_{ñ}^{+}$(ƨ), $Đ_{ñ}^{-}$(ƨ)): ƨ∈R} = ((ξ1, ξ2, ξ3, ξ4), (ξ0, ξ2, ξ3, ξ5)) is called a trapezoidal intuitionistic fuzzy number (TZ_{IFN}) if

$$Đ_{ñ}^{+} = \begin{cases} \frac{x - \xi_1}{\xi_2 - \xi_1}, \xi_1 \le x \le \xi_2 \\ 1, \xi_2 \le x \le \xi_3 \\ \frac{\xi_4 - x}{\xi_4 - \xi_3}, \xi_3 \le x \le \xi_4 \\ 0, otherwise \end{cases}$$

and

$$Đ_{ñ}^{-}=\begin{cases}\frac{\xi_2 - x}{\xi_2-\xi_0}, \xi_0 \le x \le \xi_2 \\ 0, \xi_2 \le x \le \xi_3 \\ \frac{x-\xi_3}{\xi_5-\xi_3}, \xi_3 \le x \le \xi_5 \\ 1, otherwise\end{cases}$$

Definition 2.27. [5] Intuitionistic fuzzy arithmetical operations are defined for addition, subtraction, multiplication and division of trapezoidal intuitionistic fuzzy numbers.

Let Ñ = $((\varsigma_1,\varsigma_2,\varsigma_3,\varsigma_4),(\varsigma_0,\varsigma_2,\varsigma_3,\varsigma_5))$ and Û = $((\xi_1, \xi_2, \xi_3, \xi_4), (\xi_0, \xi_2, \xi_3, \xi_5))$ be two TZ_{IFN}s. Then

(i) the addition of Ñ and Û is Ñ + Û = $((\varsigma_1+\xi_1, \varsigma_2+\xi_2, \varsigma_3+\xi_3, \varsigma_4+\xi_4), (\varsigma_0+\xi_0, \varsigma_2+\xi_2, \varsigma_3+\xi_3, \varsigma_5+\xi_5))$,

(ii) The multiplication of Ñ and Û is Ñ×Û = $((\varsigma_1 \xi_1, \varsigma_2 \xi_2, \varsigma_3 \xi_3, \varsigma_4 \xi_4), (\varsigma_0 \xi_0, \varsigma_2 \xi_2, \varsigma_3 \xi_3, \varsigma_5 \xi_5))$, where $\varsigma_0, \varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5, \xi_0, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ are all non zero positive real numbers,

(iii) –Û = $((-\xi_4, -\xi_3, -\xi_2, -\xi_1), (-\xi_5, -\xi_3, -\xi_2, -\xi_0))$, the subtraction of Û from Ñ is Ñ–Û = $((\varsigma_1- \xi_4, \varsigma_2-\xi_3, \varsigma_3- \xi_2, \varsigma_4- \xi_1), (\varsigma_0- \xi_5, \varsigma_2- \xi_3, \varsigma_3- \xi_2, \varsigma_5- \xi_0))$,

(iv) $\frac{1}{\hat{U}}=\hat{U}^{-1}=((\frac{1}{\xi_4},\frac{1}{\xi_3},\frac{1}{\xi_2},\frac{1}{\xi_1}),(\frac{1}{\xi_5},\frac{1}{\xi_3},\frac{1}{\xi_2},\frac{1}{\xi_0}))$, and the division of Ñ and Û is $\frac{\hat{N}}{\hat{U}}=((\frac{\varsigma_1}{\xi_4},\frac{\varsigma_2}{\xi_3},\frac{\varsigma_3}{\xi_2},\frac{\varsigma_4}{\xi_1}),(\frac{\varsigma_0}{\xi_5},\frac{\varsigma_2}{\xi_3},\frac{\varsigma_3}{\xi_2},\frac{\varsigma_5}{\xi_0}))$ where $\xi_0, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ are all non zero positive real numbers,

(v) For any real number Ǩ,

ǨÑ = $((Ǩ\varsigma_1, Ǩ\varsigma_2, Ǩ\varsigma_3, Ǩ\varsigma_4),(Ǩ\varsigma_0, Ǩ\varsigma_2, Ǩ\varsigma_3, Ǩ\varsigma_5))$ if Ǩ > 0

ǨÑ = $((Ǩ\varsigma_4, Ǩ\varsigma_3, Ǩ\varsigma_2, Ǩ\varsigma_1),(Ǩ\varsigma_5, Ǩ\varsigma_3, Ǩ\varsigma_2, Ǩ\varsigma_0))$ if Ǩ < 0.

Definition 2.28. (i). Intuitionistic defuzzification of trapezoidal intuitionistic fuzzy numberÑ = $((a_1, a_2, a_3, a_4), (a_0, a_2, a_3, a_5))$ can be done by the signed distance(SD). It is defined as

$$d(\hat{N},0)=S(\hat{N})=\frac{1}{4}\int_0^1[a_1+l(a_2-a_1)+a_4-l(a_4-a_3)+a_2-(1-l)(a_2-a_0)+a_3+(1-l)(a_5-a_3)]dl$$

$$=\frac{a_1+a_2+a_3+a_4+a_0+a_2+a_3+a_5}{8}=\frac{a_0+a_1+2a_2+2a_3+a_4+a_5}{8}.$$

(ii). TIVM of Ñ is defined as

$$TI(\hat{N})=\frac{1}{2}\int_0^1[a_1+l(a_2-a_1)+a_4-l(a_4-a_3)+a_2-(1-l)(a_2-a_0)+a_3+(1-l)(a_5-a_3)]dl$$

$$=\frac{a_0+a_1+2a_2+2a_3+a_4+a_5}{4}.$$

3. INTUITIONISTIC FUZZY NUMBER LABELING GRAPH

Definition 3.1. An intuitionistic fuzzy number graceful labeling($_{IFN}$GfL) of a $_{IF}$G $\Omega = (\Phi, \Theta, \mathfrak{J})$ having q $_{IF}$Es is an injective mapping $\mathfrak{L} : \Phi(\Omega) \rightarrow \{\tilde{0}, \tilde{1}, \tilde{2}, \ldots, \tilde{q}\}$ such that when each $_{IF}$E$\zeta\varsigma$ is assigned the label of the difference of $\mathfrak{L}(\zeta)$ and $\mathfrak{L}(\varsigma)$, the resulting $_{IF}$E labels are distinct. An intuitionistic fuzzy number graceful graph($_{IFN}$GfG) is one which admits a $_{IFN}$GfL.

Theorem 3.2. Every $_{IF}P_a$ is a $_{IFN}$GfG.

Proof. Let W_m be a $_{IF}P_a$ with m $_{IF}$Vs. That is W_m has the number of $_{IF}$Es is m-1. Labeling can begin at either end without loss of generality. The first point at one end is labeled as $\tilde{0}$, the adjacent point is labeled as $\widetilde{m-1}$,the next adjacent, non labeled point is labeled as $\tilde{1}$ and we continue in this manner. Alternate points are incrementally increasing by 1 while the remaining points are incrementally decreasing by 1. Let s = $[\frac{m}{2}]$. For cases when m is even, the $_{IF}$E labels beginning with the leftmost edge in figure are | $(\widetilde{m-1}) - \tilde{0}$|, | $(\widetilde{m-1}) - \tilde{1}$|,..., | $(\widetilde{m-s}) - \widetilde{s-1}$|, all are distinct.

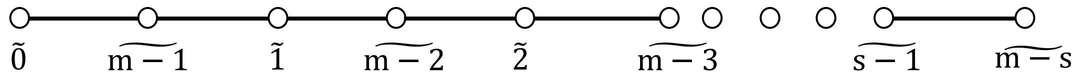

Fig. 3.1. W_m for m is even

For cases when m is odd, the edge labels begining with the leftmost edge in figure are | $(\widetilde{m-1}) - \tilde{0}$|, | $(\widetilde{m-1}) - \tilde{1}$|,..., | $(\widetilde{m-s}) - \tilde{s}$|, all are distinct.

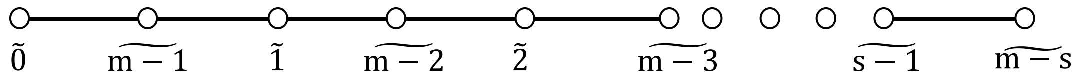

Fig. 3.2. Wm for m is odd

Remark 3.3. (i) Every crisp graceful path is a $_{IFN}$Gf path graph.

(ii) Every $_{FN}$Gf path graph is a $_{IFN}$Gf path graph.

Example 3.4. A triangular intuitionistic fuzzy number path graph($_{TIFN}P_a$G) with 6 and 7 $_{IF}$Vs.

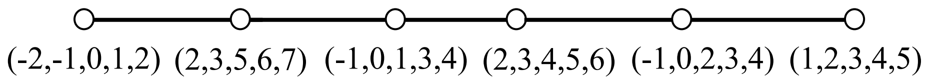

Fig. 3.3. A $_{TIFN}P_a$G with 6 $_{IF}$Vs

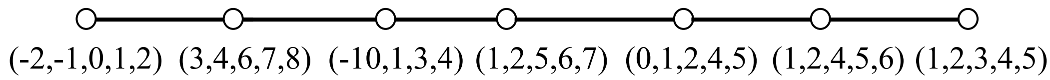

Fig. 3.4. A $_{TIFN}P_a$G with 7 $_{IF}$Vs

Example 3.5. A trapezoidal intuitionistic fuzzy number path graph($_{TPIFN}P_a$G) with 6 and 7 $_{IF}$Vs.

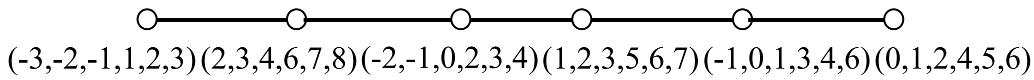

Fig. 3.5. A $_{TPIFN}P_a$G with 6 $_{IF}$Vs

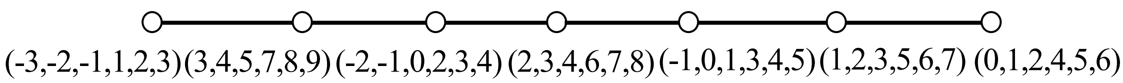

Fig. 3.6. A $_{TPIFN}P_a$G with 7 $_{IF}$Vs

Example 3.6. A Hexagonal intuitionistic fuzzy number path graph($_{HIFN}P_aG$) with 6 and 7 $_{IF}$Vs.

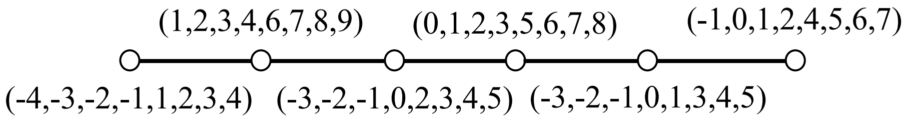

Fig. 3.7. A $_{HIFN}P_aG$ with 6 $_{IF}$Vs

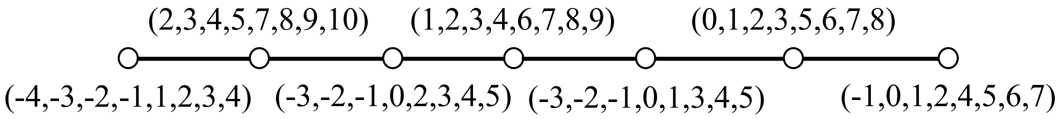

Fig. 3.8. A $_{HIFN}P_aG$ with 7 $_{IF}$Vs

Proposition 3.7. $_{IF}C_{om}G$ K_2 is a $_{IFN}$Gf complete graph.

Proof. From the $_{IF}G$ K_2 is $_{IFN}$Gf complete graph.

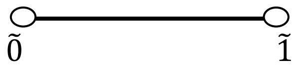

Fig. 3.9

Example 3.8. $_{TIFN}G$ complete graph K_2.

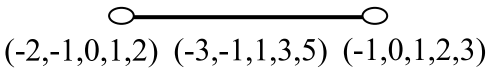

Fig. 3.10

Example 3.9. Trapezoidal $_{IFN}G$ complete graph K_2.

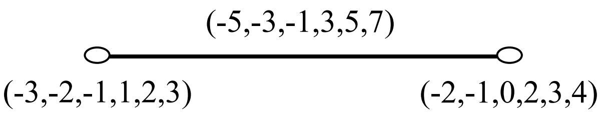

Fig. 3.11

Example 3.10. Hexagonal $_{IFN}G$ complete graph K_2.

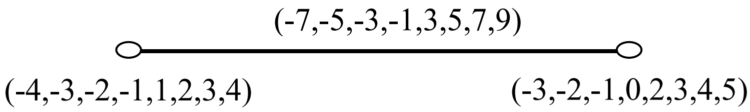

Fig. 3.12

Proposition 3.11. $_{IF}C_{om}G$ K_3 is a $_{IFN}$Gf complete graph.

Proof. From the $_{IF}G$, K_3 is $_{IFN}$Gf complete graph.

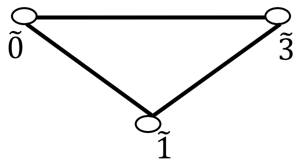

Fig. 3.13

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Example 3.12. TIFNGf complete graph K3.

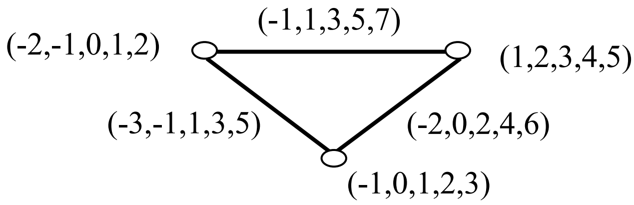

Fig. 3.14

Example 3.13. Trapezoidal IFNGf complete graph K3.

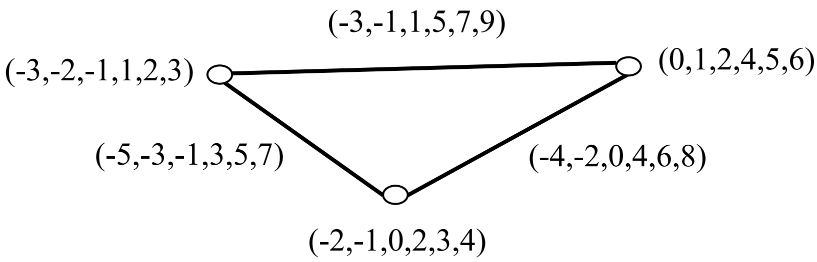

Fig. 3.15

Example 3.14. Hexagonal IFNGf complete graph K3.

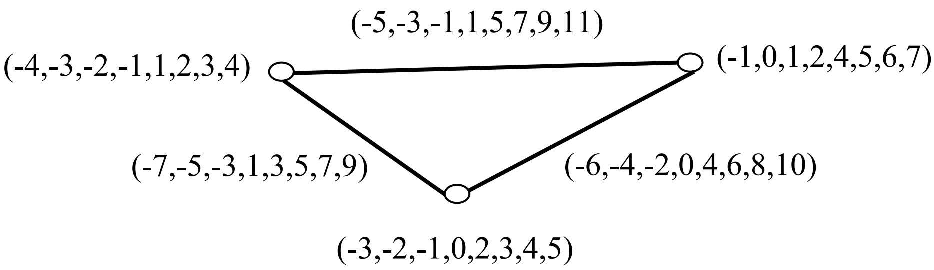

Fig. 3.16

Proposition 3.15. IFComG K4 is IFNGf complete graph.

Proof. From the IFG, K4 is a IFNGf complete graph.

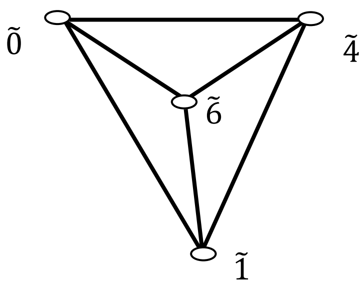

Fig. 3.17

Example 3.16. TIFNGf complete graph K4.

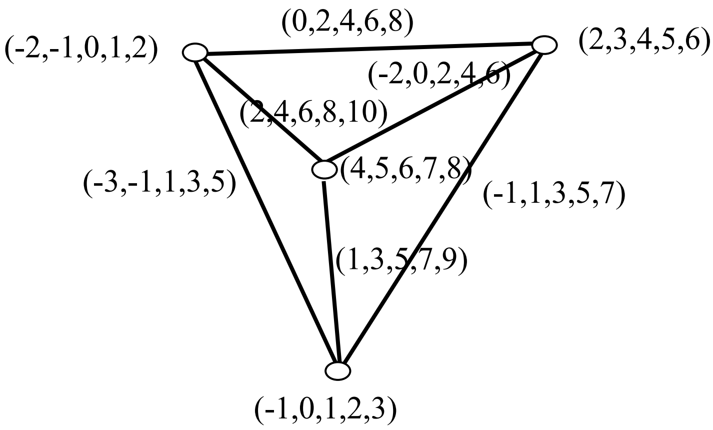

Fig. 3.18

Example 3.17. Trapezoidal $_{IFN}$Gf complete graph K_4.

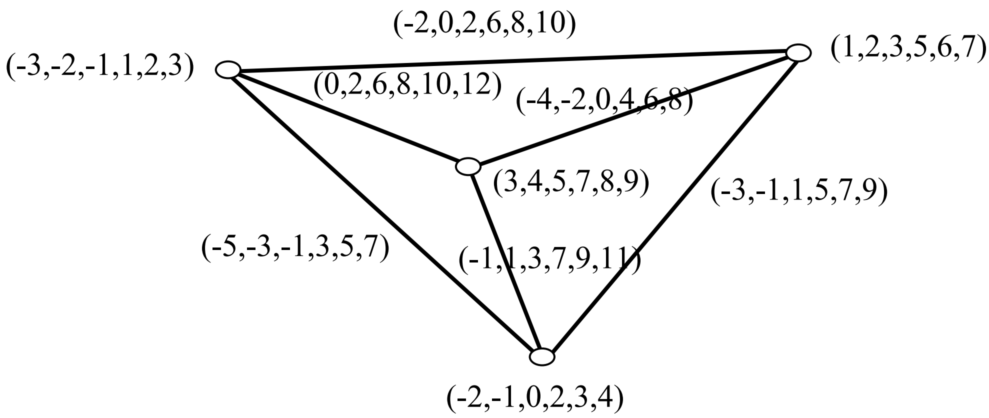

Fig. 3.19

Example 3.18. Hexagonal $_{IFN}$Gf complete graph K_4.

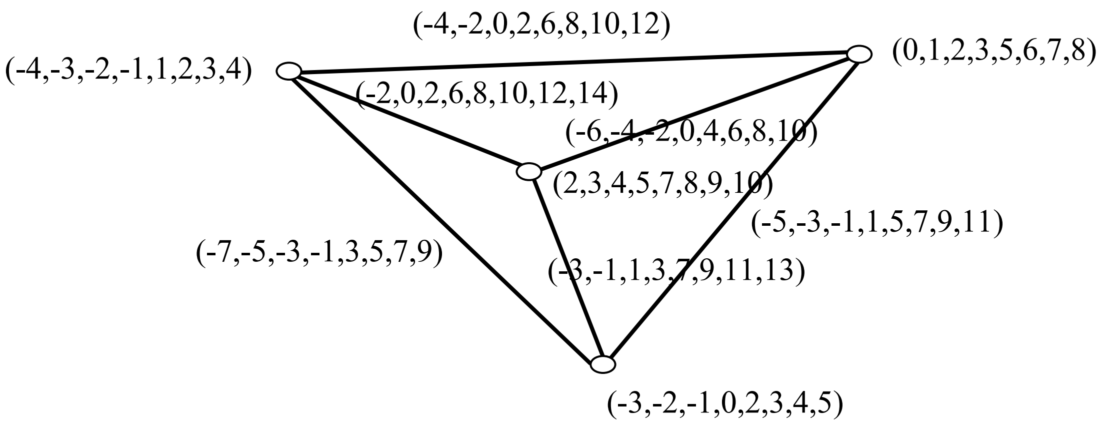

Fig. 3.20

Proposition 3.19. $_{IF}C_{om}G$ K_5 is a $_{IFN}$Gf complete graph.

Proof. From the $_{IF}$G, K_5 is $_{IFN}$Gf complete graph.

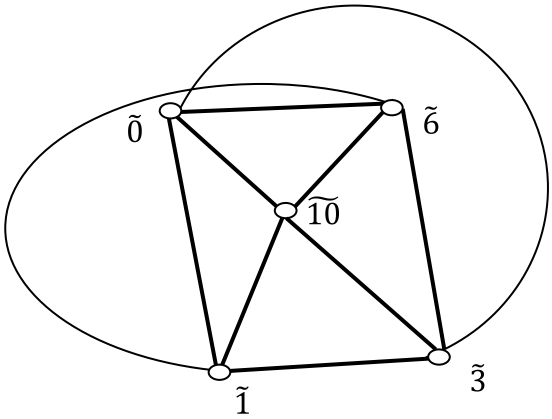

Fig. 3.21

Example 3.20. TIFNGf complete graph K_5.

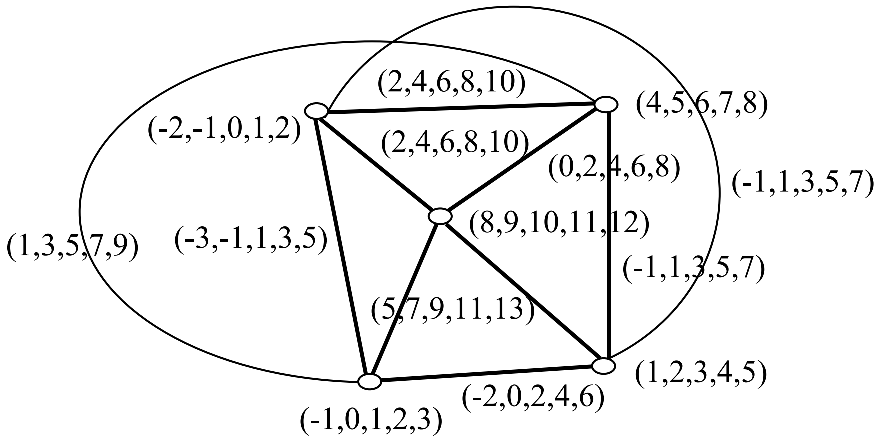

Fig. 3.22

Example 3.21. Trapezoidal IFNGf complete graph K_5.

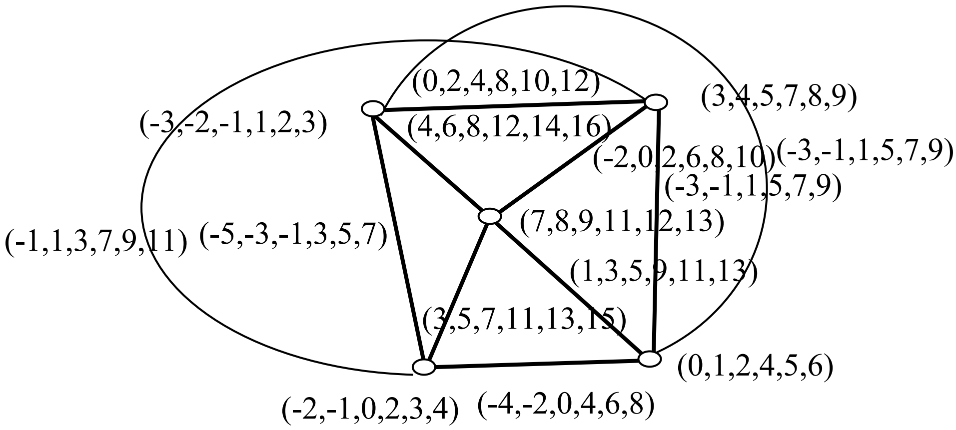

Fig. 3.23

Example 3.22. Hexagonal IFNGf complete graph K_5.

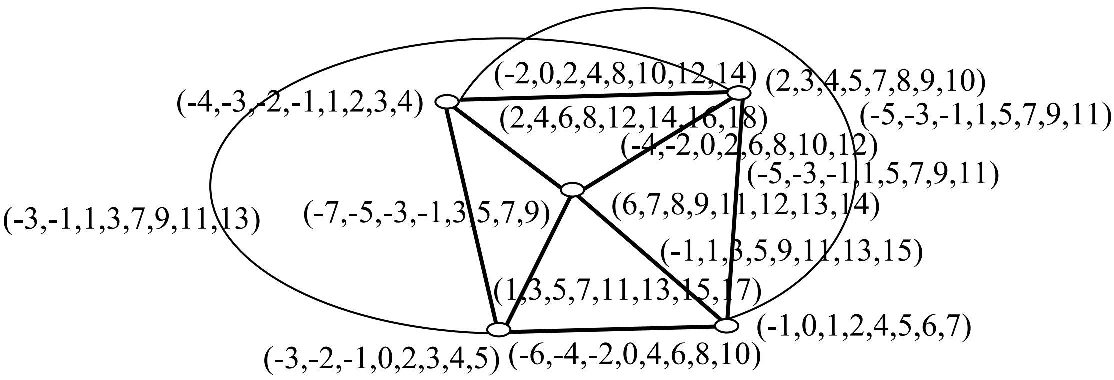

Fig. 3.24

Remark 3.23. In crisp graph, K_5 is not a graceful complete graph.

Proposition 3.24. $_{IF}C_{om}G$ K_6 is a $_{IFN}Gf$ complete graph.

Proof. From the $_{IF}G$, K_6 is $_{IFN}Gf$ complete graph.

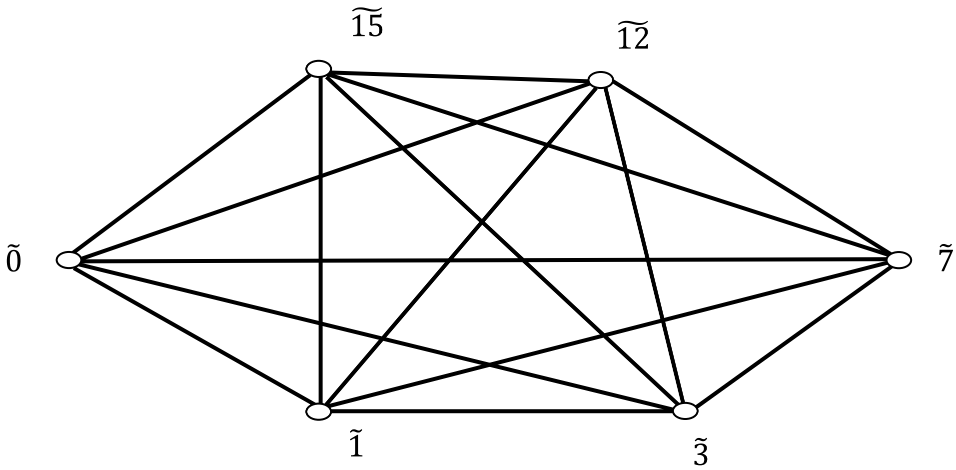

Fig. 3.25

Example 3.25. $_{TIFN}Gf$ complete graph K_6.

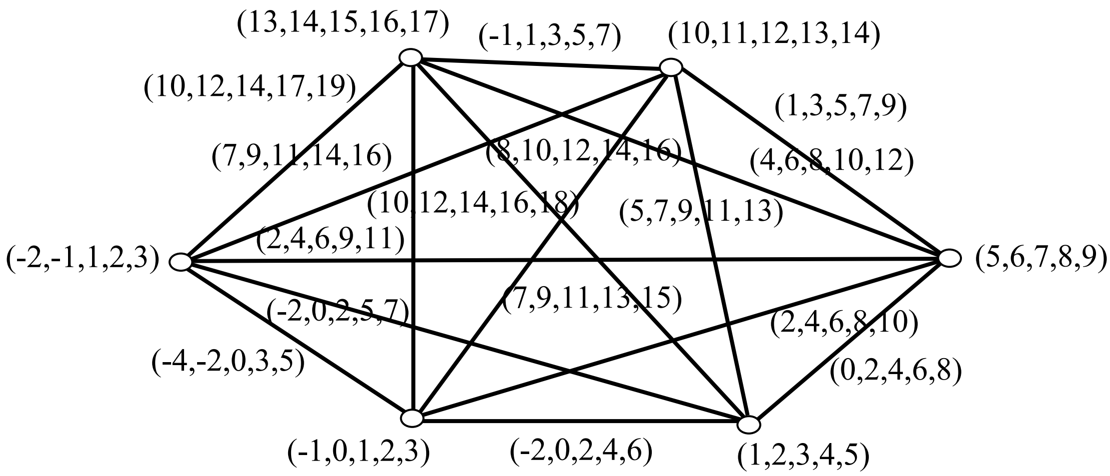

Fig. 3.26

Example 3.26. Trapezoidal $_{IFN}$Gf complete graph K_6.

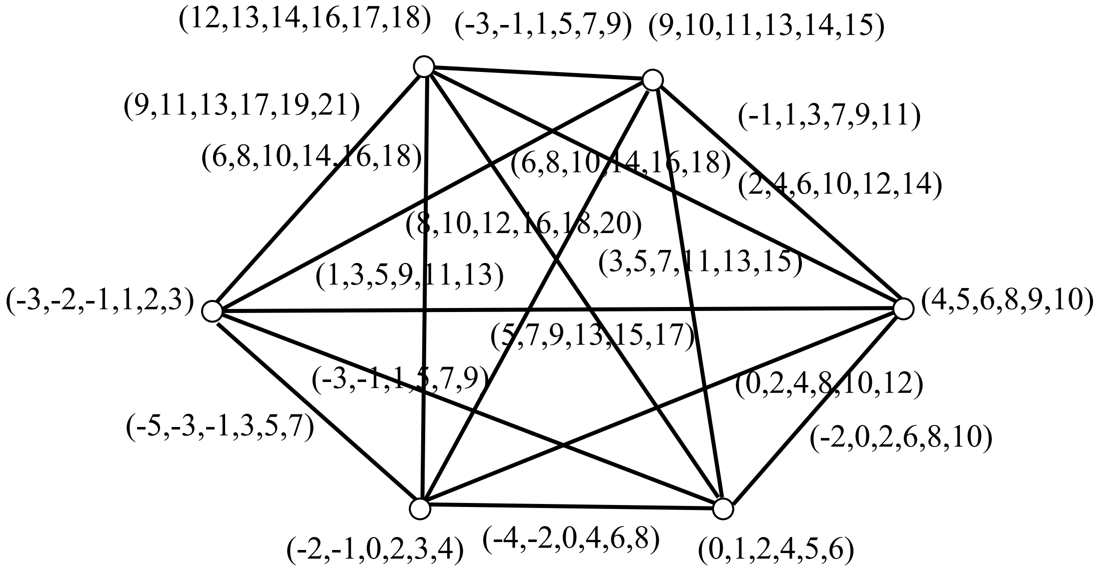

Fig. 3.27

Example 3.27. Hexagonal $_{IFN}$Gf complete graph K_6.

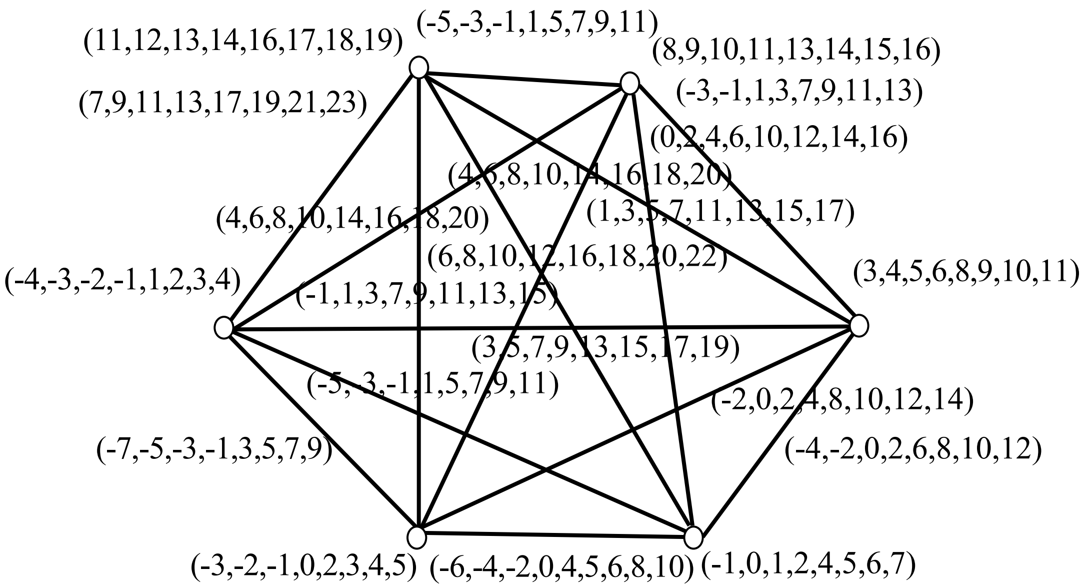

Fig. 3.28

Remark 3.28. In crisp graph, K_6 is not a graceful complete graph.

Proposition 3.29. $_{IF}C_{om}G$ K_7 is $_{IFN}$Gf complete graph.

Proof. From the $_{IF}G$ figure, K_7 is $_{IFN}$Gf complete graph.

s

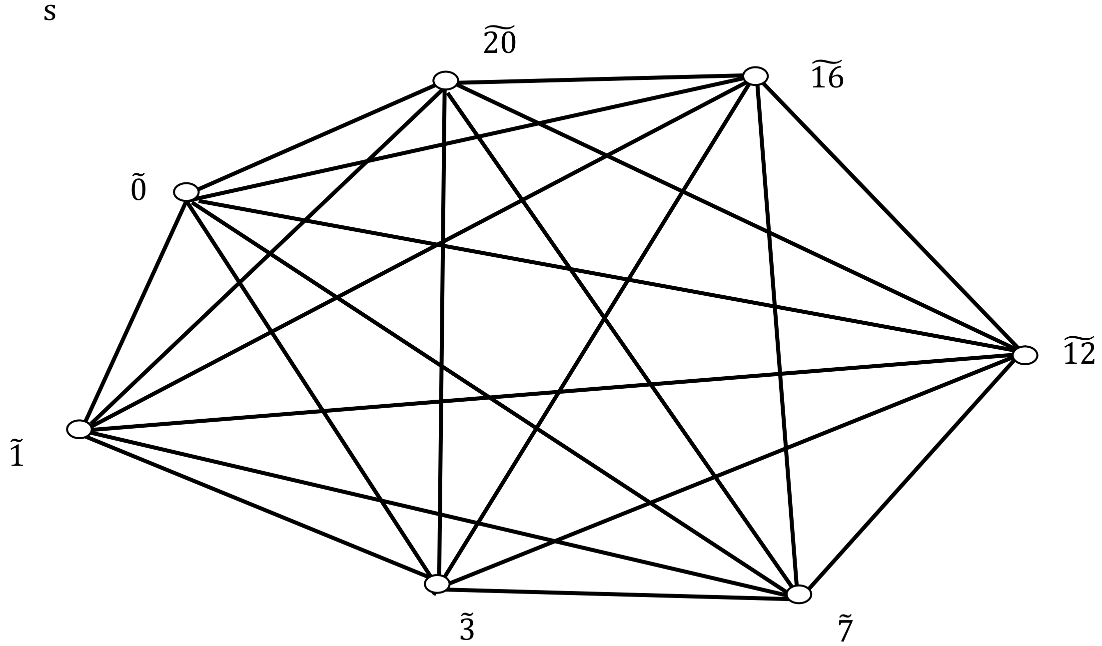

Fig. 3.29

Example 3.30. $_{TIFN}$Gf complete graph K_7.

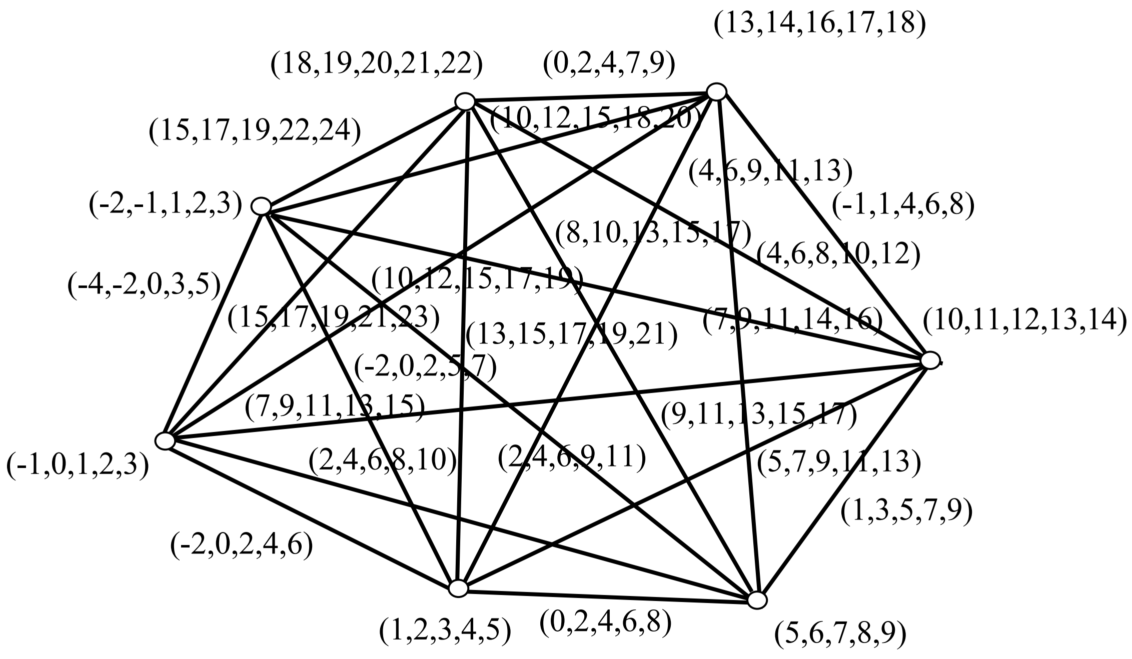

Fig. 3.30

Example 3.31. Trapezoidal $_{IFN}$Gf complete graph K_7.

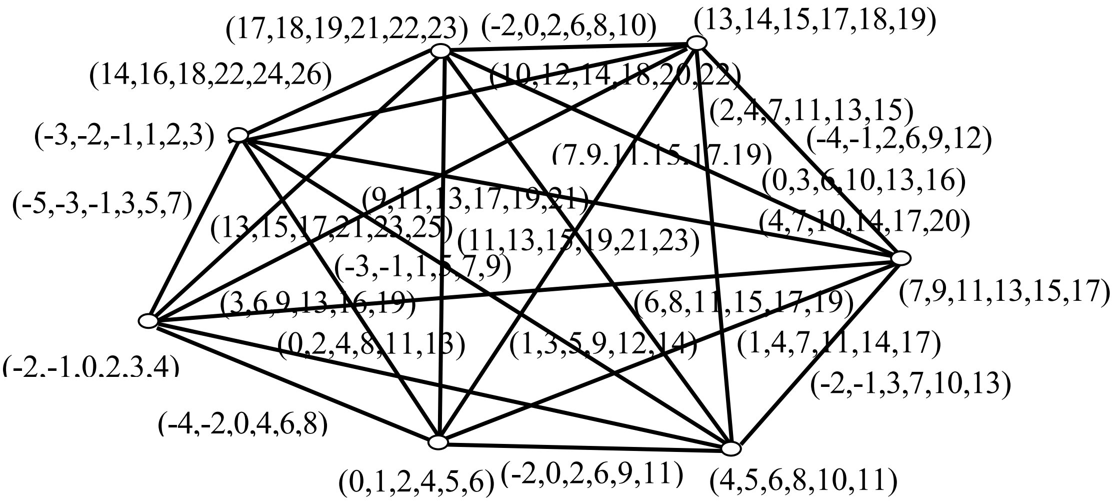

Fig. 3.31

Example 3.32. Hexagonal $_{IFN}$Gf complete graph K_7.

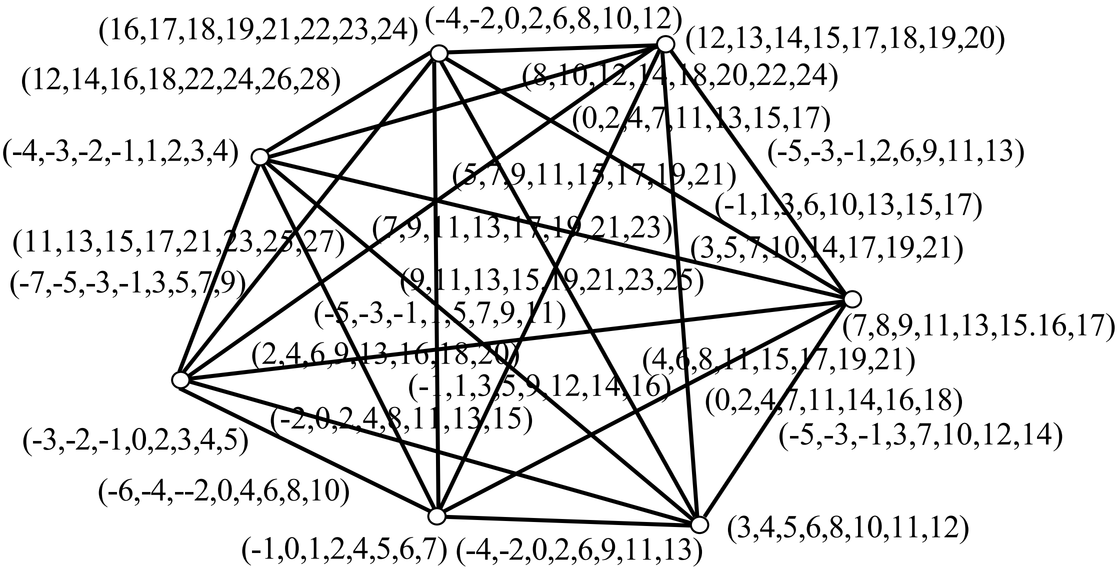

Fig. 3.32

Remark 3.33. In crisp graph, K_7 is not graceful complete graph.

Proposition 3.34. Every $_{IF}C_{om}G$ $\Omega = (\Phi, \Theta, \Im)$ is $_{IFN}$Gf complete graph.

Proof. From the generalization of the above proposition, Ω is a $_{IFN}$Gf complete graph.

Proposition 3.35. $_{IF}C_y$ C_3 is $_{IFN}$Gf cycle graph.

Proof. From the $_{IF}$G figure, C_3 is $_{IFN}$Gf cycle graph.

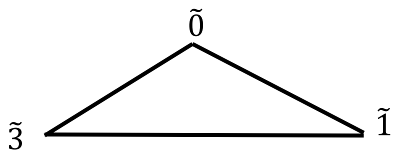

Fig. 3.33

Example 3.36. $_{\text{TIFN}}$Gf cycle graph C_3.

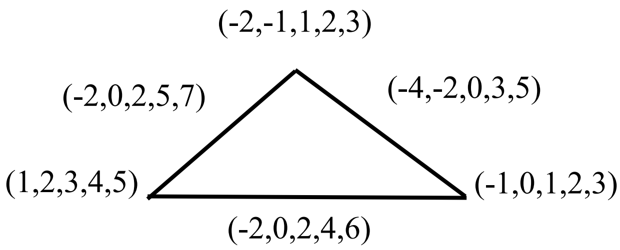

Fig. 3.34

Example 3.37. Trapezoidal $_{\text{IFN}}$Gf cycle graph C_3.

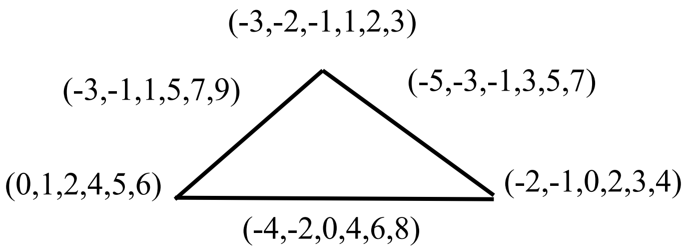

Fig. 3.35

Example 3.38. Hexagonal $_{\text{IFN}}$Gf cycle graph C_3.

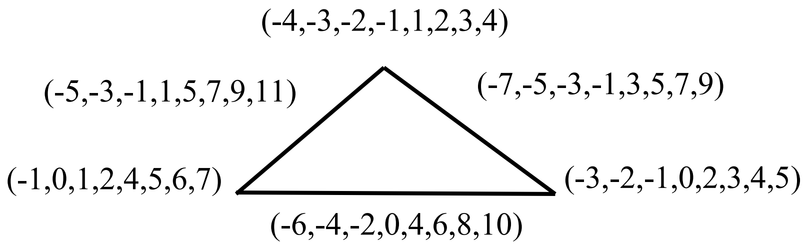

Fig. 3.36

Proposition 3.39. $_{\text{IF}}$C$_y$ C_4 is a $_{\text{IFN}}$Gf cycle graph.

Proof. From the $_{\text{IF}}$G figure, C_4 is a $_{\text{IFN}}$Gf cycle graph.

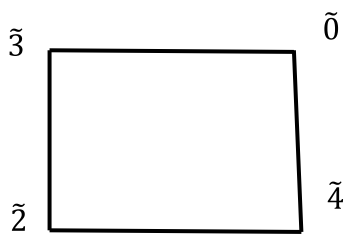

Fig. 3.37

Example 3.40. $_{\text{TIFN}}$Gf cycle graph C_4.

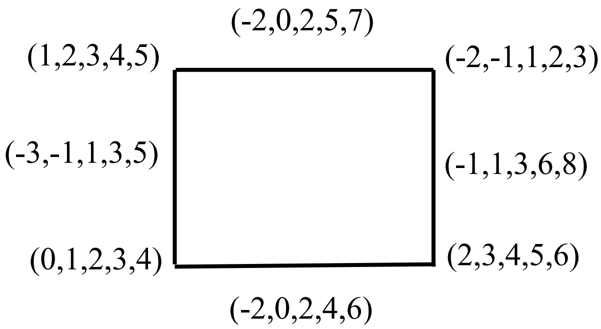

Fig. 3.38

Example 3.41. Trapezoidal $_{IFN}$Gf cycle graph C_4

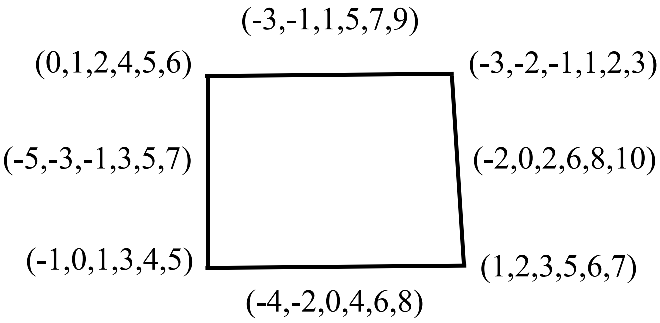

Fig. 3.39

Example 3.42. Hexagonal $_{IFN}$Gf cycle graph C_4.

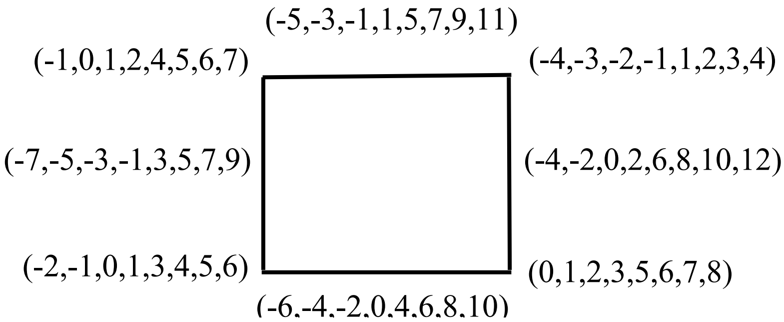

Fig. 3.40

CONCLUSION

Intuitionistic fuzzy number graceful labeling is one of the main branch in intuitionistic fuzzy graph. Here different types of graceful labeling definition is given, and some theorems are stated and proved. Using the above definitions and theorems, we can find more results. It can be extended into different types of intuitionistic fuzzy number graceful labeling.

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